Four-loop verification of algorithm for Feynman diagrams summation in $N=1$ supersymmetric electrodynamics.

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Abstract

A method of Feynman diagrams summation, based on using Schwinger-Dyson equations and Ward identities, is verified by calculating some four-loop diagrams in $N=1$ supersymmetric electrodynamics, regularized by higher derivatives. In particular, for the considered diagrams correctness of an additional identity for Green functions, which is not reduced to the gauge Ward identity, is proved.

1 Introduction.

Current indirect proofs of existence of the supersymmetry in the Standard model make the problem of calculation of quantum corrections in supersymmetric theories especially urgent. Due to the supersymmetry the ultraviolet behavior of field theory models is essentially improved. For example even in theories with unextended supersymmetry it is possible to suggest the form of the β-function exactly to all orders of the perturbation theory. In the case of $N=1$ supersymmetric electrodynamics, which will be considered in this paper, this β-function (that is called the exact Novikov, Shifman, Vainshtein and Zakharov (NSVZ) β-function) is [1]:

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)\Big), \tag{1}$$

where $\gamma(\alpha)$ is the anomalous dimension of the matter superfield.

Explicit calculations, made with the dimensional reduction [2], confirm this proposal, but require a special choice of the subtraction scheme [3, 4]. Explicit calculations in two- [5, 6] and three-loop [7] approximations for the $N=1$ supersymmetric

electrodynamics with the higher derivative regularization [8, 9] reveal that renormalization of the operator $W_a C^{ab} W_b$ is exhausted at the one-loop and the Gell-Mann-Low function coincides with the exact NSVZ β-function and has corrections in all orders of the perturbation theory.

An attempt to perform explicit calculations exactly to all orders of the perturbation theory was made in Ref. [10]. According to this paper it is possible to calculate a large number of Feynman diagrams exactly to all orders of the perturbation theory using Schwinger-Dyson equations and Ward identities. Nevertheless, some diagrams can not be obtained by this method. However, explicit calculations (up to the three-loop approximation inclusive) show, that the sum of undefined contributions is 0. This means existence of an identity, which is not reduced to the Ward identities and can be graphically written in the form (31). (In the analitic form this identity is given by Eq. (30)). It relates the two-point Green function of a matter superfield and a three-point Green function, in which external lines correspond to a gauge field, a chiral matter superfield and an external source introduced in a special way. This source is an arbitrary scalar superfield.

Identity (30) is important because it restricts nontrivially the structure of the divergences of the theory. Really, due to the supersymmetry and gauge invariance the two-point Green function of the gauge field should be proportional to $V\partial^2\Pi_{1/2}V$, the coefficient being an unknown function. (It is a supersymmetric analog of the transversality condition in the ordinary electrodynamics.) Using the Schwinger-Dyson equations and Ward identities it is possible to rewrite this function via the two-point Green function of the matter superfields and another function, which is not fixed by the gauge invariance. Actually this means, that the two-point Green function of the gauge superfield is equivalently rewritten in a different form, as earlier, up to an undefined (from the Ward identities or, equivalently, from the gauge invariance) function. However, according to new identity the terms, which depend on the unknown function, are actually equal to zero. So this identity really removes the arbitrariness in the structure of the divergence.

Certainly, it is highly desirable to find a true reason of this identity – for example, some symmetry. But first it is necessary to verify, that it really takes place and is not caused by an accidental coincidence.

The considered identity is nontrivial starting from the three-loop approximation. In Ref. [11] it was proved for a special class of diagrams exactly to all orders of the perturbation theory. Such diagrams contain the only loop of the matter superfields and any two cuts of this loop do not make the diagram disconnected. (In this case the technical part of the proof is simpler.) Possibly in the general case the proof can be made similarly. In order to confirm that such a proof is possible, it is desirable to verify identity (30) for other diagrams. This requires making explicit four-loop calculations. These calculations are made in this paper for a special group of four-loop diagrams, which can be made disconnected by two cuts of the matter superfields loop.

The paper is organized as follows:

In Sec. 2 the basic information about $N = 1$ supersymmetric electrodynamics and its regularization by higher derivatives is reminded. The method of summation Feynman diagrams, based on using Schwinger-Dyson equations and Ward identities is

described in Sec. 3. In this section we write an identity, to which the Green functions are supposed to satisfy. Four-loop verification of the results, presented in Sec. 3, is made in Sec. 4. A brief summary of the paper is presented in the Conclusion.

2 $N=1$ supersymmetric electrodynamics and higher derivative regularization.

The massless $N=1$ supersymmetric electrodynamics with the higher derivatives term in the superspace is described by the following action:

$$S=\frac{1}{4e^2}\mathrm{Re}\int d^4x\,d^2\theta\,W_a C^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b+\frac{1}{4}\int d^4x\,d^4\theta\,\Big(\phi^* e^{2V}\phi+\tilde\phi^* e^{-2V}\tilde\phi\Big). \quad (2)$$

Here ϕ and $\tilde\phi$ are the chiral matter superfields and V is a real scalar superfield, which contains the gauge field A_μ as a component. The superfield W_a is a supersymmetric analog of the stress tensor of the gauge field. In the Abelian case it is defined by

$$W_a=\frac{1}{16}\bar D(1-\gamma_5)D\Big[(1+\gamma_5)D_a V\Big], \quad (3)$$

where D is a supersymmetric covariant derivative. It is important to note, that in the Abelian case the superfield W^a is gauge invariant, so that action (2) will be also gauge invariant.

Quantization of model (2) can be made by the standard way. For this purpose it is convenient to use the supergraphs technique, described in book [12] in details, and to fix the gauge invariance by adding the following terms:

$$S_{gf}=-\frac{1}{64e^2}\int d^4x\,d^4\theta\,\bigg(V D^2\bar D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V+V\bar D^2 D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V\bigg), \quad (4)$$

where

$$D^2\equiv\frac{1}{2}\bar D(1+\gamma_5)D;\qquad \bar D^2\equiv\frac{1}{2}\bar D(1-\gamma_5)D. \quad (5)$$

After adding such terms a part of the action, quadratic in the superfield V, will have the simplest form

$$S_{gauge}+S_{gf}=\frac{1}{4e^2}\int d^4x\,d^4\theta\,V\partial^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V. \quad (6)$$

In the Abelian case, considered here, diagrams containing ghost loops are absent.

It is well known, that adding of the higher derivative term does not remove divergences in one-loop diagrams. In order to regularize them, it is necessary to insert in the generating functional the Pauli-Villars determinants [13].

The generating functional can be written in the form

$$Z = \int DV\, D\phi\, D\tilde\phi\, \prod_i \Big(\det PV(V,M_i)\Big)^{c_i} \exp\Big(i(S_{ren}+S_{gf}+S_S+S_{\phi_0})\Big). \qquad (7)$$

Here

$$S_{ren} = \frac{1}{4e^2} Z_3(e,\Lambda/\mu)\,\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b + \\ +Z(e,\Lambda/\mu)\,\frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi+\tilde\phi^* e^{-2V}\tilde\phi\Big) \qquad (8)$$

is the renormalized action, the action for the gauge fixing terms are given by Eq. (4) (It is convenient to substitute e by the bare charge e_0 in it), the Pauli-Villars determinants are defined by

$$\Big(\det PV(V,M)\Big)^{-1} = \int D\Phi\, D\tilde\Phi\, \exp\Big(iS_{PV}\Big), \qquad (9)$$

where

$$S_{PV} \equiv Z(e,\Lambda/\mu)\frac{1}{4}\int d^4x\, d^4\theta\,\Big(\Phi^* e^{2V}\Phi+\tilde\Phi^* e^{-2V}\tilde\Phi\Big) + \\ +\frac{1}{2}\int d^4x\, d^2\theta\, M\tilde\Phi\Phi + \frac{1}{2}\int d^4x\, d^2\bar\theta\, M\tilde\Phi^*\Phi^*, \qquad (10)$$

and the coefficients c_i satisfy conditions

$$\sum_i c_i = 1; \qquad \sum_i c_i M_i^2 = 0. \qquad (11)$$

Below we will assume, that $M_i = a_i\Lambda$, where a_i are some constants. Insertion of Pauli-Villars determinants allows to cancel remaining divergences in all one-loop diagrams, including diagrams, containing insertions of counterterms.

The terms with sources are written in the form

$$S_S = \int d^4x\, d^4\theta\, JV + \int d^4x\, d^2\theta\,\Big(j\,\phi+\tilde j\,\tilde\phi\Big) + \int d^4x\, d^2\bar\theta\,\Big(j^*\phi^*+\tilde j^*\tilde\phi^*\Big). \quad (12)$$

Moreover, in generating functional (7) we introduced the expression

$$S_{\phi_0} = \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi_0^*\, e^{2V}\phi+\phi^*\, e^{2V}\phi_0+\tilde\phi_0^*\, e^{-2V}\tilde\phi+\tilde\phi^*\, e^{-2V}\tilde\phi_0\Big), \qquad (13)$$

where ϕ_0, ϕ_0^*, $\tilde\phi_0$ and $\tilde\phi_0^*$ are scalar superfields. They are some parameters, which are not chiral or antichiral. In principle, it is not necessary to introduce the term S_{ϕ_0} into

the generating functional, but the presence of the parameters ϕ_0 is highly desirable for the investigation of Schwinger-Dyson equations.

In our notations the generating functional for the connected Green functions is written as

$$W = -i \ln Z, \tag{14}$$

and an effective action is obtained by making a Legendre transformation:

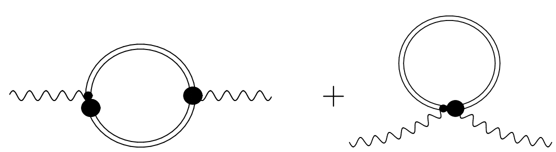
$$\Gamma = W - \int d^4x\, d^4\theta\, JV - \int d^4x\, d^2\theta \left(j\,\phi + \tilde{j}\,\tilde{\phi} \right) - \int d^4x\, d^2\bar{\theta} \left(j^*\phi^* + \tilde{j}^*\tilde{\phi}^* \right), \tag{15}$$

where the sources J, j and $\tilde{j}$ is to be eliminated in terms of the fields V, ϕ and $\tilde{\phi}$, through solving equations

$$V = \frac{\delta W}{\delta J}; \qquad \phi = \frac{\delta W}{\delta j}; \qquad \tilde{\phi} = \frac{\delta W}{\delta \tilde{j}}. \tag{16}$$

3 Summation of Feynman diagrams in $N = 1$ supersymmetric quantum electrodynamics

From generating functional (7) it is possible to obtain [10] the Schwinger-Dyson equations, which are graphically written as

$\Gamma^{(2)}_V =$  (17)

where $\Gamma^{(2)}_V$ is a two-point Green function of the gauge field. We will always set the renormalization constant $Z = 1$, because the dependence on Z can be easily restored in the final result and this dependence is not essential in this paper.

The double line in Eq. (17) denotes the exact propagator ($Z = 1$)

$$\left(\frac{\delta^2\Gamma}{\delta\phi^*_x \delta\phi_y} \right)^{-1} = -\frac{D^2_x \bar{D}^2_x}{4\partial^2 G} \delta^8_{xy}, \tag{18}$$

in which the function $G(q^2)$ is defined by the two-point Green function as follows:

$$\frac{\delta^2\Gamma}{\delta\phi^*_x \delta\phi_y} = \frac{D^2_x \bar{D}^2_x}{16} G(\partial^2) \delta^8_{xy}, \tag{19}$$

where $\delta^8_{xy} \equiv \delta^4(x-y)\delta^4(\theta_x - \theta_y)$, and the lower indexes denote points, in which considered expressions are taken.

The large circle denotes the effective vertex, which is written as [10]

$$\left.\frac{\delta^3\Gamma}{\delta V_x\delta\phi_y\delta\phi_z^*}\right|_{p=0} = \partial^2\Pi_{1/2x}\Big(\bar D_x^2\delta^8_{xy}D_x^2\delta^8_{xz}\Big)F(q^2) + \\ +\frac{1}{32}q^\mu G'(q^2)\bar D\gamma^\mu\gamma_5 D_x\Big(\bar D_x^2\delta^8_{xy}D_x^2\delta^8_{xz}\Big) + \frac{1}{8}\bar D_x^2\delta^8_{xy}D_x^2\delta^8_{xz}\,G(q^2), \quad (20)$$

due to the Ward identities. The strokes denote derivatives with respect to q^2,

$$\Pi_{1/2} = -\frac{1}{16\partial^2}D^a\bar D^2 D_a = -\frac{1}{16\partial^2}\bar D^a D^2\bar D_a \quad (21)$$

is a supersymmetric transversal projector, and $F(q^2)$ is a function, which can not be defined from the Ward identities. Here

$$D^a \equiv \Big[\frac{1}{2}\bar D(1+\gamma_5)\Big]^a; \qquad D_a \equiv \Big[\frac{1}{2}(1+\gamma_5)D\Big]_a; \\ \bar D^a \equiv \Big[\frac{1}{2}\bar D(1-\gamma_5)\Big]^a; \qquad \bar D_a \equiv \Big[\frac{1}{2}(1-\gamma_5)D\Big]_a. \quad (22)$$

Two adjacent circles denote an effective vertex, consisting of 1PI diagrams, in which one of the external lines is attached to the very left edge. Such vertexes are given by [10]

$$\frac{\delta^2\Gamma}{\delta\phi_y\delta\phi_{0z}^*} = \frac{1}{4}\frac{\delta}{\delta\phi_y}\exp\left(\frac{2}{i}\frac{\delta}{\delta J_z}+2V_z\right)\phi_z = -\frac{1}{8}G\bar D_y^2\delta^8_{yz} \quad (23)$$

in the case of one external V-line (the vertex in the first diagram of Eq. (17)) and

$$\left.\frac{\delta^3\Gamma}{\delta V_x\delta\phi_y\delta\phi_{0z}^*}\right|_{p=0} = \frac{1}{4}\frac{\delta}{\delta V_x}\frac{\delta}{\delta\phi_y}\exp\left(\frac{2}{i}\frac{\delta}{\delta J_z}+2V_z\right)\phi_z\Big|_{p=0} = \\ = -2\partial^2\Pi_{1/2x}\Big(\bar D_x^2\delta^8_{xy}\delta^8_{xz}\Big)F(q^2) + \frac{1}{8}D^a C_{ab}\bar D_x^2\Big(\bar D_x^2\delta^8_{xy}D_x^b\delta^8_{xz}\Big)f(q^2) + \\ -\frac{1}{16}q^\mu G'(q^2)\bar D\gamma^\mu\gamma_5 D_x\Big(\bar D_x^2\delta^8_{xy}\delta^8_{xz}\Big) - \frac{1}{4}\bar D_x^2\delta^8_{xy}\delta^8_{xz}\,G(q^2), \quad (24)$$

in the case of two external V-lines (in the second diagram of Eq. (17)). Here $f(q^2)$ is one more function, which can not be found from the Ward identity.

The expression

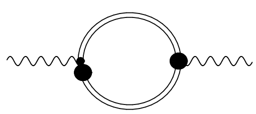
$$\left.\frac{d}{d\ln\Lambda}\Gamma^{(2)}_V\right|_{p=0}, \quad (25)$$

can be calculated [10] by substituting solutions of Ward identities into the Schwinger-Dyson equations.

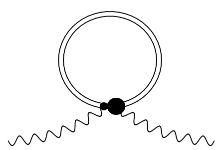
A result of calculation of any diagram can be written in the form

$$\int \frac{d^4p}{(2\pi)^4}\, d^4\theta \left(V\partial^2\Pi_{1/2}V A(p^2) + V^2 B(p^2)\right), \tag{26}$$

where A and B are some functions. It is actually (with a small modification) a supersymmetric analog of expansion to the longitudinal and transversal parts. Due to the Ward identity the sum of diagrams contains only transversal parts, i.e. terms, proportional to $V\partial^2\Pi_{1/2}V$. It is confirmed by calculations, made in Ref. [10]: Terms, proportional to V^2, are 0 in the sum of diagrams, and we will not write them here. Terms, proportional to $V\partial^2\Pi_{1/2}V$, in the effective diagrams are given by



$$= V\partial^2\Pi_{1/2}V \frac{d}{d\ln\Lambda}\int \frac{d^4q}{(2\pi)^4}\left(\frac{8F}{q^2G} + \frac{1}{2q^2}\frac{d}{dq^2}\ln\left(q^2G^2\right)\right); \tag{27}$$



$$= V\partial^2\Pi_{1/2}V \frac{d}{d\ln\Lambda}\int \frac{d^4q}{(2\pi)^4}\left(-\frac{8F}{q^2G} - \frac{8f}{q^2G}\right). \tag{28}$$

(Here we omit integration with respect to the external momentum and external θ.) Expressions for these diagrams also contain terms, proportional to V^2 and similar terms with the Pauli-Villars fields, which we omitted for brevity. Their form can be found in Ref. [10]. Terms with the Pauli-Villars fields give additional contributions, which cancel the remaining divergences. Investigation of them is a bit more complicated and requires making similar calculations in the massive theory. We will always assume existence of terms with the Pauli-Villars fields, but will not write them explicitly.

Note now, that terms, containing the unknown function F, are completely cancelled in the sum of contributions (27) and (28). However there are terms, containing the unknown function f:

$$\frac{d}{d\ln\Lambda}\frac{\delta^2\Delta\Gamma}{\delta V_x\,\delta V_y}\bigg|_{p=0} = 2\partial^2\Pi_{1/2}\delta^8_{xy}\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{2q^2}\frac{d}{dq^2}\ln(q^2G^2) - \frac{8f}{q^2G} - (PV)\right), \tag{29}$$

where (PV) denotes similar terms with the Pauli-Villars fields. Nevertheless, explicit three-loop calculations show that the following identity takes place:

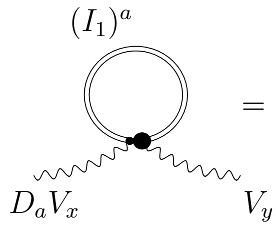
$$\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{f(q^2)}{q^2G(q^2)} = 0, \tag{30}$$

where the function G is defined from the two-point Green function of the matter superfield by Eq. (19), and the function f – from the three-point Green function with the external lines V, ϕ and ϕ_0^* by Eq. (24).

An attempt to prove identity (30) exactly to all orders was made in Ref. [11] by approximately the same method, which is used for the proof of the Ward identity diagram by diagram. Although application of this method appears to be possible, the proof was not made in the general case due to the technical difficulties. It was made

only for diagrams, which contain the only loop of the matter superfields and remain connected after any two cuts of this loop. In the general case the proof, made by the same method, seems to be possible, but much more complicated technically.

Identity (30) can be graphically written as



$$= 0, \qquad (31)$$

that was also proved in Ref. [11]. The symbol $(I_1)^a$ (here we use notations of Ref. [11]) means, that the double line in this case corresponds to the expression

$$-\frac{1}{2\partial^2 G(\partial^2)} D^a_x \bar{D}^2_x \delta^8_{xz}, \qquad (32)$$

instead of the effective propagator (18), which is proportional to $D^2_x \bar{D}^2_x \delta^8_{xz}$. Note, that equality (30) is not valid in the massive case. Its corresponding modification can be found in [10].

4 Four-loop calculations

Note now, that expressions (27) and (28) allow to find not only sums of all diagrams with two external lines of the gauge field, but also sums of special classes of such diagrams. Such classes of diagrams are obtained from a frame, to which external lines are attached by all possible ways. For example, in Fig. 1 we present a frame for diagrams, which are investigated in this paper. The dots denote all possible points, to which two external lines of the gauge field can be attached.

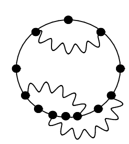


Figure 1: Schematic picture of the class of diagrams, considered in the paper

If a frame contains the only loop of the matter superfield and can not be made disconnected by two cuts of this loop, then for the corresponding class of diagrams identity (31) was proved in Ref. [11]. However it is desirable to verify, if this identity takes place in the other cases. For this purpose we consider diagrams, which are obtained from the frame, presented in Fig. 1, by adding two external lines. (Such diagrams can be evidently made disconnected by two cuts of the loop of matter superfields.)

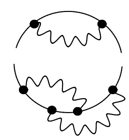
We will calculate the considered diagrams by two ways:

1. using Eqs. (27) and (28) and the functions G, f and F, obtained preliminary.

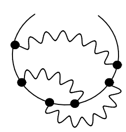
2. by explicit calculation using the supergraph method.

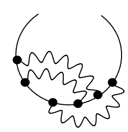
Thus it is possible to perform a four-loop verification of Eqs. (27) and (28), and also identity (30).

In order to find diagrams which will be essential for finding the unknown functions G, f and F in the considered case, it is convenient to use the following simple speculations: A diagram, presented in Fig. 1, can be considered as a formal product of one- and two-loop diagrams with ϕ^* and ϕ external lines (pairs of their ends are identified):



or as a three-loop diagram with the identified ends:





The parts of the diagram, obtained by this way, are the Feynman diagrams for finding the function G. To define the functions f and F it is necessary to add one more external line of the superfield V to such diagrams.

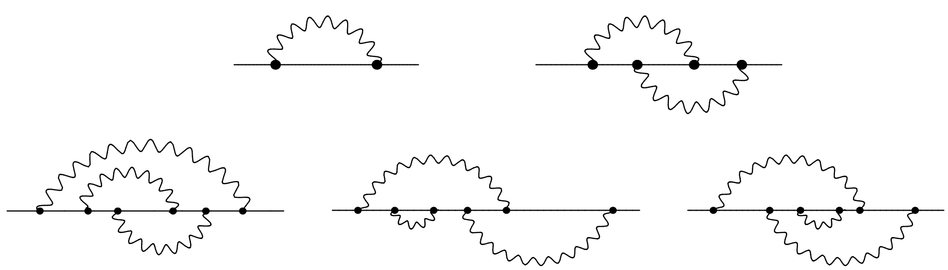


Figure 2: The diagrams, used for calculation of the function G

Thus it order to find the function G in the considered case it is necessary to calculate diagrams presented in Fig. 2. Then the function G is obtained according to definition (19). One-, two- and three-loop parts of the function G, defined by diagrams in Fig. 2, will be denoted by G_1, G_2 and G_3 respectively. Expressions for them are presented in Appendix A. The complete function G (certainly without diagrams, which are not essential for this paper) in the considered approximation is given by

$$G(q^2) = 1 + G_1 + G_2 + G_3, \tag{33}$$

where the unity is a tree contribution.

The functions $F(q^2)$ and $f(q^2)$ are defined from three-loop Green functions by Eq. (24). As in the case of the function G, in order to obtain the contribution, which

corresponds to the considered class of the four-loop diagrams, it is sufficient to calculate only diagrams of a special form. These diagrams are obtained from diagrams, presented in Fig. 2, by all possible insertions of one external V-line.

We will denote one-, two- and three-loop contributions to the function f by f_1, f_2 and f_3 respectively. Similar notation we will use for the function F. Because in the tree approximation these functions are 0 and $f_1 = 0$, in the considered order we have

$$F = F_1 + F_2 + F_3; \qquad f = f_2 + f_3. \tag{34}$$

Expressions for f_1, f_2, f_3, F_1 and F_2, obtained by calculation of the above pointed diagrams, are presented in Appendix A. An expression for F_3 has been calculated, but it is not presented because it is very large. (It is not required for verification of identity (30).)

Using the obtained expressions it is possible to verify identity (30). With the considered accuracy we have

$$\frac{f}{q^2 G} = \frac{1}{q^2}\Big(f_3 - f_2 \cdot G_1\Big). \tag{35}$$

Substituting here expressions for the functions G_1, f_2 and f_3 from Eqs. (39), (43), (44) we obtain, that the integrand can be written as a total derivative with respect to the momentum q:

$$\int \frac{d^4q}{(2\pi)^4}\,\frac{f}{q^2G} = ie^6 \int \frac{d^4q\,d^4k\,d^4l\,d^4r}{(2\pi)^{16}} \left[1+(-1)^n\frac{k^{2n}}{\Lambda^{2n}}\right]^{-1} \left[1+(-1)^n\frac{l^{2n}}{\Lambda^{2n}}\right]^{-1} \times$$
$$\times \left[1+(-1)^n\frac{r^{2n}}{\Lambda^{2n}}\right]^{-1} \frac{\partial}{\partial q^\mu}\left\{\frac{(2q+k+l)^\mu}{k^2l^2r^2q^2(k+q)^2(q+l)^2(q+r)^2(k+q+l)^2}\right\}. \tag{36}$$

This equality can be verified by calculating the derivative with respect to q^μ using the Leibnitz rule and comparing the result with Eq. (35), in which expressions for the functions G_1, f_2 and f_3 are obtained by explicit calculation of diagrams.

Because the integrand in Eq. (36) is a total derivative with respect to q^μ of the expression, which goes to 0 in the limit $q \to \infty$, expression (36) is 0. This means that identity (30) is correct in the considered approximation and for the considered class of diagrams.

Note, that in this case using of the higher derivative regularization is essential. In spite of the parameter Λ is encountered only in propagators of the superfield V, which do not contain the momentum q, this regularization allows to make calculations in the limit $p \to 0$. It follows from existence of limit (25), which was proved in Ref. [10]. Taking this limit is senseless without the higher derivative regularization, because this produces infinities. Another important reason of using the higher derivative regularization is the possible problems with integration with respect to the momentum q in Eq. (29) in $D \neq 4$ dimensions. For example, it is possible, that in D dimensions expression (35) can not be presented as a total derivative. In principle, identity (30) can be considered as a formal relation in the non-regularized theory, which is made sensible by the regularization. However, most likely it is not valid for an arbitrary regularization in

the regularized theory. Because supersymmetric theories are usually regularized either by the dimensional reduction or by the higher derivative regularization, it is sensible to say, that identity (30) is valid in the theory, regularized by higher derivatives.

It is also desirable to verify the method of summation of Feynman diagrams by using the Schwinger-Dyson equations and Ward identities. For this purpose it is possible to calculate both effective diagrams in Eq. (17) in the four-loop approximation explicitly and compare the result with Eqs. (27) and (28). In considered approximation

$$\frac{1}{q^2}\frac{d}{dq^2}\ln(G^2) = \frac{q^\mu}{q^4}\frac{\partial}{\partial q^\mu}\Big(G_3 - G_1\cdot G_2\Big); \tag{37}$$

$$\frac{F}{q^2 G} = \frac{1}{q^2}\Big(F_3 - F_2\cdot G_1 - F_1\cdot G_2\Big). \tag{38}$$

Explicit calculations were made by the method, proposed in Ref. [14], which simplifies finding a part of diagram, proportional to $V\partial^2\Pi_{1/2}V$. Nevertheless, it does not allow to obtain a part, proportional to V^2. That is why the verification of Eqs. (27) and (28) was made only for a part, proportional to $V\partial^2\Pi_{1/2}V$. In the both cases this verification completely confirms them.

As a small technical remark let us note, that making this verification it was necessary to take into account, that a large number of ordinary Feynman diagrams contributed both to the first effective diagram in Eq. (17) and to the second one. For example, it is easy to see, that a contribution of diagram, presented in Fig. 3

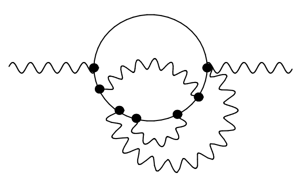


Figure 3: One of the diagrams, giving contribution to both effective diagrams

is divided into parts 3/4 and 1/4, which correspond to the first and to the second diagrams in Eq. (17).

5 Conclusion

In this paper the four-loop verification of identity (30) has been made for a special class of Feynman diagrams. The proof, presented in Ref. [11], is not applicable for these diagrams, because structure of the diagrams sum is more complicated and there are additional restrictions to possible positions of the gauge field external lines. Due to the confirmation of the considered identity, obtained here, this identity seems to be also valid for arbitrary diagrams. Its origin seems to be a symmetry of the theory. However, this symmetry surely is not the gauge invariance.

To conclude we stress again, that the especial importance of the obtained identity is that it essentially restricts the form of divergences in the considered theory. The matter is that it is possible to reduce the expression for the two-point Green function of the gauge field to the expression, depending only on the two-point Green function of the matter superfield and LHS of this identity, using only the gauge invariance. Thus the considered identity removes all undefined expressions from the two-point Green function of the gauge field and relates it with the two-point Green function for the matter superfield.

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A Explicit expressions for the functions G, f and F

Below we present expressions for the functions G, f and F. The lower index shows in what order of the perturbation theory (for diagrams of the considered class) the corresponding function was calculated.

$$G_1 = 2\,ie^2 \int \frac{d^4k}{(2\pi)^4}\,\Big[1+(-1)^n\frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\frac{1}{k^2\,(q+k)^2}; \qquad (39)$$

$$G_2 = 4\,e^4 \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\Big[1+(-1)^n\frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\Big[1+(-1)^n\frac{l^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\times \qquad (40)$$

$$\times\frac{(2q+k+l)^2}{k^2\,l^2(k+q)^2\,(q+l)^2\,(k+q+l)^2};$$

$$G_3 = -4\,ie^6 \int \frac{d^4k\,d^4l\,d^4r}{(2\pi)^{12}}\,\Big[1+(-1)^n\frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\Big[1+(-1)^n\frac{l^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\times$$

$$\times\Big[1+(-1)^n\frac{r^{2n}}{\Lambda^{2n}}\Big]^{-1}\,\frac{1}{k^2l^2r^2(q+k)^2(q+l)^2(q+k+l)^2}\times$$

$$\times\Bigg(\frac{(2q+2k+r+l)^2(q+l)^2}{(q+r+k)^2\,(q+r+k+l)^2}+\frac{(2q+k+l)^2}{(q+r+k+l)^2}+\frac{2(2q+k+l)^2}{(q+r+k)^2}\Bigg). \qquad (41)$$

$$f_1(q^2) = 0; \tag{42}$$

$$f_2(q^2) = \frac{1}{2}\, e^4 \int \frac{d^4k\, d^4l}{(2\pi)^8}\, \Big[1+(-1)^n \frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1} \Big[1+(-1)^n \frac{l^{2n}}{\Lambda^{2n}}\Big]^{-1} \times$$
$$\times \frac{1}{k^2\, l^2\, (k+q)^2\, (q+l)^2 (k+q+l)^2} \times$$
$$\times \Bigg(-2 + \frac{(2q+k+l)_\mu (k+q)^\mu}{(k+q)^2} + \frac{(2q+k+l)_\mu (l+q)^\mu}{(l+q)^2} \Bigg); \tag{43}$$

$$f_3(q^2) = -4i\, e^6 \int \frac{d^4k\, d^4l\, d^4r}{(2\pi)^{12}}\, \Big[1+(-1)^n \frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1} \Big[1+(-1)^n \frac{l^{2n}}{\Lambda^{2n}}\Big]^{-1} \times$$
$$\times \Big[1+(-1)^n \frac{r^{2n}}{\Lambda^{2n}}\Big]^{-1} \frac{1}{k^2\, l^2\, r^2\, (k+q)^2\, (q+l)^2\, (q+k+l)^2} \Bigg(\frac{1}{(k+q+r)^2} +$$
$$+ \frac{1}{2(k+l+q+r)^2} - \frac{(2q+k+l)^\mu (k+q)^\mu}{2(k+q)^2\, (k+q+r)^2} - \frac{(2q+k+l)^\mu (l+q)^\mu}{2(l+q)^2\, (k+q+r)^2} -$$
$$- \frac{(2q+k+l)^\mu (k+q)^\mu}{2(k+q)^2\, (k+l+q+r)^2} - \frac{(2q+k+l)^\mu (k+q+r)^\mu}{2(k+q+r)^4} \Bigg). \tag{44}$$

$$F_1(q^2) = -\frac{ie^2}{8} \int \frac{d^4k}{(2\pi)^4}\, \Big[1+(-1)^n \frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1} \frac{1}{k^2\, (k+q)^4}; \tag{45}$$

$$F_2(q^2) = \frac{1}{4}\, e^4 \int \frac{d^4k\, d^4l}{(2\pi)^8}\, \Big[1+(-1)^n \frac{k^{2n}}{\Lambda^{2n}}\Big]^{-1} \Big[1+(-1)^n \frac{l^{2n}}{\Lambda^{2n}}\Big]^{-1} \times$$
$$\times \frac{1}{k^2\, l^2\, (k+q)^2\, (q+l)^2 (k+q+l)^2} \Bigg(4 - 2\, \frac{(q+l)^2}{(k+q)^2} - \frac{(2q+k+l)^2}{(q+k+l)^2} \Bigg). \tag{46}$$

(An expression for F_3 is not presented, because it is too large.)

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